

PARTICULATE DIFFUSION ACROSS A PLANE TURBULENT JET

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Abstract—The fundamental equations of a two-dimensional turbulent jet without pressure gradient in the lateral direction are used to derive a similarity solution for particulate mass transfer across an air curtain.

The analysis also applies to heat transfer across the jet. The solution, valid for the fully developed region, was derived using the constant eddy diffusivity hypothesis. The analysis can be applied to spatially varying concentrations on one side of the jet.

Experimental work with dusts confirms the model. An important result is that the development of a concentration or temperature profile take more time than the development of the velocity profile. The fully developed concentration profile occurs at the minimum distance of twenty times the nozzle thickness.

Finally the formula for accumulation of particles in the filtering system is derived for a steady concentration difference. Applications of the results are discussed.

NOMENCLATURE

A ,	cross-sectional area of a jet;
A_q ,	turbulent diffusivity;
b ,	jet width;
b_0 ,	nozzle width;
b_s ,	b at $x = s$;
c, k ,	constants;
cx ,	mixing length;
D_T ,	$= A_q/\rho$;
$F(\eta)$,	flow function;
J ,	momentum of a jet;
k ,	kinematic momentum of a jet;
L ,	fixed distance on x -axis;
m ,	$= 2Sch_T$;
Pr_T ,	turbulent Prandtl number;
s ,	fixed value of x ;
Sch_T ,	turbulent Schmidt number;
u ,	time average jet velocity in x -direction;
u' ,	fluctuating velocity in x -direction;
\bar{u} ,	average of jet velocity over the collecting area;
$u_o = u_o(x)$,	centerline velocity of the jet;
u_{max} ,	outlet velocity of the jet;
u_s ,	u_o at $x = s$;
v ,	average jet velocity in y -direction;
v' ,	fluctuating velocity in y -direction;
V ,	average particle volume;
W ,	mass of dust collected;
x ,	vertical jet coordinate;
y ,	horizontal jet coordinate.

Greek symbols

ε ,	virtual kinematic viscosity;
ε_s ,	outlet kinematic viscosity;
η ,	$= \sigma y/x$ —dimensionless jet variable;
$\theta(\eta)$,	function of concentration across a jet;
μ ,	viscosity;
ξ ,	concentration of aerosol;
ξ_o, ξ_{max} ,	maximum dust concentration in the chamber;
ξ_{min} ,	minimum concentration;
ρ ,	fluid density;
σ ,	$= 7.67$ —experimental constant;
τ ,	shearing stress;
χ_1 ,	experimental constant;
ψ ,	stream function.

INTRODUCTION

THE SUBJECT of this study is the diffusion of aerosol particles across a turbulent jet. The analysis developed applies to the fully developed region of the jet only. Experiments have been performed, which show that the particle concentration across the jet can be described by a hyperbolic tangent function, as predicted by the analysis. Particle diffusion is complex because of the possibility of variation of the Schmidt number with particle size.

One application of the study is the possibility of isolating a region of contaminating dust with an air curtain. Air curtains are two-dimensional jets in various design arrangements. The fluid mechanics of jets have been investigated by many researchers with various hypotheses used. Reviews of this work are given in Abramovich [1] and Schlichting [2]. The present work is based upon the classical analysis of Goertler as

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reviewed in Schlichting. Other investigators have treated various aspects of the air curtain problem. For example, Niitsu and Katoh [3] have considered the velocity distribution and deflection of jets subject to lateral side pressure and with lateral side stream. Their work also includes correlations of gas infiltration across a jet. Further investigations have been done on air curtains as means of reducing heat flux across a doorway. The work preceding the modern air curtain heat diffusion was done on heated jets with solutions by Abramovich and others.

As in other turbulent flows, the analogy between heat and mass diffusion will be assumed in the case of the two-dimensional turbulent jet. The heated jet surrounded on both sides by a constant temperature has been treated by Abramovich with the experimental work by Borodachev [1], who also studied concentration profiles of a carbon dioxide jet issuing into air. Abramovich concluded that these two problems are identical in nature and can be described by the same analytic model. Hayes [4] obtained experimental results on a full scale spill type air curtain and supported it by a finite difference computer simulation technique. The finite difference solution worked out by Hayes extends the solution over a "transition" region of $6 \leq L/b_0 \leq 20$, which is neither a mixing layer, nor a fully developed region of a plane jet.

Velocity distribution

We assume that the flow is turbulent, incompressible, steady state, and contains a dust of low enough concentration that the density, ρ , is essentially that of the fluid alone.

In order to clarify the derivation of the mass concentration profile we present the Goertler solution here.

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial \tau}{\partial y} \quad (1)$$

and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

where u and v are the average velocities and the notation is shown on Fig. 1: ρ is the density of the fluid and τ denotes the shearing stress

$$\tau = \mu \frac{\partial u}{\partial y} - \rho \overline{u'v'}$$

where u' , v' are the fluctuating components of the velocity vectors. Adopting Prandtl's form for the shearing stress as outlined in Schlichting

$$\tau = \rho \varepsilon \frac{\partial u}{\partial y} = \rho \chi_1 b u_o \frac{\partial u}{\partial y} \quad (3)$$

where u_o is the centerline velocity.

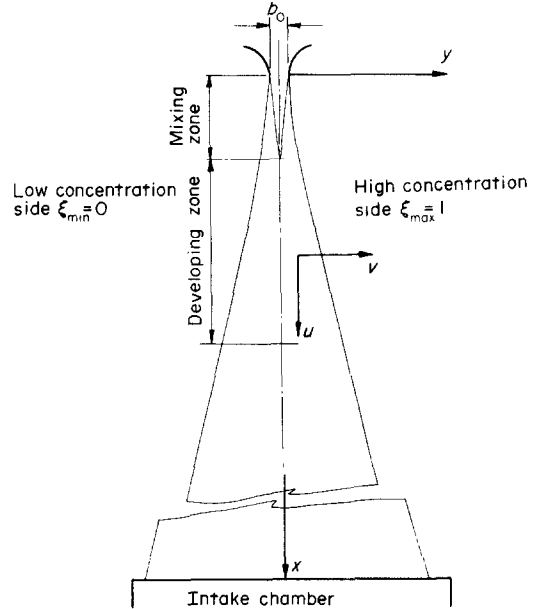


FIG. 1. Definition sketch.

$$\varepsilon = \chi_1 b u_o \quad (4)$$

is the virtual kinematic viscosity assumed to be constant over the width of the jet b and χ_1 is an experimental constant. Using the above equations, the basic differential equation for the two-dimensional turbulent jet becomes, after neglecting viscous terms:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \varepsilon \frac{\partial^2 u}{\partial y^2} \quad (5)$$

The decay of the centerline velocity can be obtained from the momentum equation

$$J = \text{const} = \rho \int_{-\infty}^{\infty} u^2 dA \quad (6)$$

where dA is an element of cross sectional area. Denoting the centerline velocity and the width of the jet at a fixed characteristic distance, s , from the orifice by u_s and b_s , respectively, we can write

$$b = b_s \frac{x}{s} \quad (7)$$

and

$$u_o = u_s \sqrt{\left(\frac{s}{x}\right)} \quad (8)$$

Consequently,

$$\varepsilon = \varepsilon_s \sqrt{\left(\frac{x}{s}\right)} \quad (9)$$

with

$$\varepsilon_s = \chi_1 b_s u_s = \text{const.} \quad (10)$$

If, as is usual in similarity analysis, we introduce a dimensionless variable, we may define:

$$\eta = \sigma \frac{y}{x} \tag{11}$$

$$\psi = \frac{1}{\sigma} u_s [\sqrt{(sx)}] F(\eta). \tag{12}$$

Substituting these derivatives into equation (5) we obtain:

$$\frac{1}{2} [F'(\eta)]^2 + \frac{1}{2} F(\eta) F''(\eta) + \varepsilon_s \frac{\sigma^2}{su_s} F'''(\eta) = 0 \tag{13}$$

confirming the existence of the postulated similarity solution.

Using equation (8) as a boundary condition at centerline $\eta = 0$

$$F''(\eta) = 0; \quad F'(\eta) = 1 \quad \text{and} \quad F(\eta) = 0$$

we can also write

$$\sigma = \frac{1}{2} \sqrt{\left(\frac{u_s s}{\varepsilon_s} \right)}. \tag{14}$$

Two integrations give the velocity profile

$$u = \frac{\sqrt{3}}{2} \cdot \left(\frac{k\sigma}{x} \right)^{1/2} (1 - \tanh^2 \eta) \tag{15}$$

$$v = \frac{\sqrt{3}}{4} \cdot \left(\frac{k}{x\sigma} \right)^{1/2} \{ 2\eta(1 - \tanh^2 \eta) - \tanh \eta \}$$

and

$$F(\eta) = \tanh \eta \tag{15a}$$

where $\sigma = 7.67$ was experimentally determined by Reichardt and $k = J/\rho$ is the kinematic momentum.

The mass concentration profile

In order to describe the case of a clean air jet separating a clean and dusty environment, we assume that the concentration of the jet fluid is the same as the lower concentration of the two fluids separated by the jet. We take as our beginning point the assumption that the basic equation for this model is

$$u \frac{\partial \xi}{\partial x} + v \frac{\partial \xi}{\partial y} = \frac{A_q}{\rho} \frac{\partial^2 \xi}{\partial y^2} \tag{16}$$

where ξ denotes concentration of the aerosol in particles/cm³ and A_q is the turbulent diffusivity. For simplicity denote

$$\frac{A_q}{\rho} = D_T. \tag{17}$$

On the basis of previous work on heated jets, where a constant Prandtl number was assumed and found to be valid, we assume a constant turbulent Schmidt number along the longitudinal axis of the jet. Since

the Schmidt number is determined as

$$\frac{\varepsilon}{D_T} = Sch_T = \text{const}, \tag{18}$$

we have on the basis of equation (9)

$$D_T = D_{TS} \sqrt{\left(\frac{x}{s} \right)}. \tag{19}$$

We now attempt to find a similarity solution to equation (16). For the concentration profile we assume

$$\xi(x, y) = g(x)\theta(\eta) \tag{20}$$

where $\xi(x, y)$ is a concentration function across the jet which would depend on the particle transfer similarity function, $\theta(\eta)$ and on the concentration variation $g(x)$ along side of the jet. Substituting equations (15), (19) and (20) into equation (16) and multiplying by a common factor

$$\frac{x}{g(x)} \cdot \frac{u_s}{\sqrt{\left(\frac{s}{x} \right)}}$$

we obtain:

$$\frac{x}{g(x)} \frac{\partial g(x)}{\partial x} F'(\eta)\theta(\eta) - \frac{1}{2} F(\eta)\theta'(\eta) = \frac{D_{TS}}{4\varepsilon_s} \theta''(\eta). \tag{21}$$

A similarity solution for the equation exists if

$$\frac{x}{g(x)} \frac{\partial g(x)}{\partial x} = k_1 \tag{22}$$

which after separation of the variables,

$$\frac{k_1}{x} dx = \frac{dg(x)}{g(x)}, \tag{23}$$

will yield a solution for the function $g(x)$:

$$g(x) = k_2 x^{k_1}. \tag{24}$$

Using this solution we can rewrite equation (21). Noting that

$$\frac{\varepsilon_s}{D_{TS}} = Sch_T,$$

we have

$$k_1 F'(\eta)\theta(\eta) - \frac{1}{2} F(\eta)\theta'(\eta) = \frac{1}{4Sch_T} \theta''(\eta). \tag{25}$$

The case appropriate to our boundary conditions is for $g(x) = \text{const}$, hence $k_1 = 0$. Under these conditions equation (25) becomes

$$\theta''(\eta) + 2Sch_T F(\eta)\theta'(\eta) = 0. \tag{26}$$

It should be noted at this point that other similarity solutions to (25) are possible. It may be shown that for $k_1 = \frac{1}{2}$, a closed form solution for (25) is possible. The solution would correspond to the boundary conditions where the concentration on one side of the jet is constant and the concentration on the other side varies as $x^{-1/2}$.

Using equation (15a) and rearranging:

$$\frac{\theta''(\eta)}{\theta(\eta)} = -2Sch_T \tanh \eta \tag{27}$$

substituting $P = \theta(\eta)$ and $dP = \theta'(\eta)$ we can write

$$P = C_1 e^{-2Sch_T \int \tanh \eta d\eta} \tag{28}$$

Performing the integration in the exponent

$$P = C_1 e^{-2Sch_T \ln \cosh \eta}$$

or

$$\theta(\eta) = C_1 (\cosh \eta)^{-2Sch_T} \tag{29}$$

Integrating equation (29) with respect to η we obtain

$$\theta(\eta) = C_1 \int (\cosh \eta)^{-2Sch_T} d\eta + C_2 \tag{30}$$

This integral can be evaluated by a series approach over two separate intervals of $\eta = (-\infty, 0)$ and $\eta = (0, \infty)$ in order to obtain two convergent series. The final general solution for any real value of the turbulent Schmidt number, using $m = 2Sch_T$ is

$$\theta(\eta) = C_1 2^m \left\{ \sum_{k=1}^{\infty} (-1)^k \frac{a_k}{(k-1)!(m+2k-2)} e^{-(m+2k-2)\eta} \right\}_{-\infty}^{\infty} + \sum_{k=1}^{\infty} (-1)^{k+1} \frac{a_k}{(k-1)!(m+2k-2)} e^{(m+2k-2)\eta} \Big|_{-\infty}^0 \tag{31}$$

where $a_1 = 1, a_k = a_{k-1}(m+k-2)$.

Using equation (20) and the above solution, the concentration profile normalized by its maximum becomes

$$\frac{\xi - \xi_{\min}}{\xi_{\max} - \xi_{\min}} = \theta(\eta) \tag{32}$$

where ξ_{\min} is the low concentration on one side of the jet. The general solution would be awkward to use.

Table 1 provides solutions for m equal to a small integer.

Table 1			
Sch_T	$\theta(\eta)$	C_1	C_2
0.5	$\arctg(Sh\eta)$	0.3166	0
1.0	$\tanh(\eta)$	0.50	0.5
1.5	$(sh\eta/2ch^2\eta) + \frac{1}{2} \arctg(Sh\eta)$	0.642	0.5

Constants C_1 and C_2 were determined from boundary conditions at $\eta = \pm \infty$

$$\begin{aligned} \eta = \infty; & \quad \theta(\eta) = 1; & \quad \theta'(\eta) = 0 \\ \eta = -\infty; & \quad \theta(\eta) = 0; & \quad \theta'(\eta) = 0. \end{aligned}$$

The solutions for the above table are plotted in Fig. 2 along with the finite difference results of Hayes for the developing region.

The temperature profile can be solved exactly the same way, using, for example, a turbulent Prandtl number of

$$Pr_T = 0.77$$

instead of turbulent Schmidt number. ξ in this case would denote temperature.

Attempts to model the problem using other common turbulent flow models were made. For example Taylor's vorticity transport model may be reformed in terms of concentration (Abramovich [1]) to obtain:

$$u \frac{\partial \xi}{\partial x} + v \frac{\partial \xi}{\partial y} = 2c^2 x^2 \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} \frac{\partial \xi}{\partial y} \right) \tag{33}$$

where c is a constant to be determined. Adopting

$$\eta = \frac{y}{x}$$

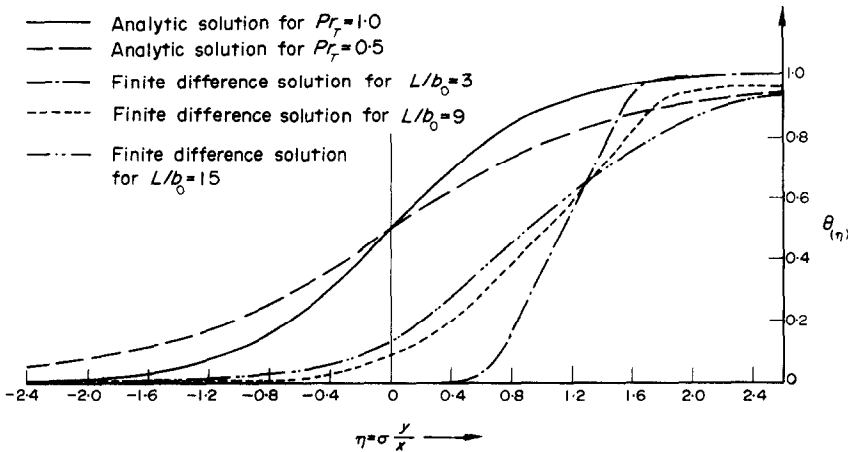


FIG. 2. Comparison of analytic solution with Hayes' finite difference temperature profiles.

as a transforming variable we obtain velocities in the form

$$u = \frac{c}{\sqrt{x}} F'(\eta) \tag{34}$$

$$v = \frac{c}{\sqrt{x}} \left[\eta F'(\eta) - \frac{1}{2} F(\eta) \right].$$

As before we assume similarity so that the concentration profile is expected to take the form

$$\xi = k\theta(\eta). \tag{35}$$

Substituting equations (34) and (35) into equation (33) and simplifying, we obtain

$$[2c^2 F''(\eta)]\theta''(\eta) + \left[\frac{1}{2} F(\eta) + 2c^2 F'''(\eta) \right] \theta'(\eta) = 0 \tag{36}$$

as the fundamental equation for this model. Separation of variables and double integration would yield

$$\theta(\eta) = - \int e^{\left[\int \frac{F'''(\eta) + \left(\frac{1}{2c}\right)^2 F(\eta)}{F''(\eta)} d\eta + C_1 \cdot d\eta \right]} + C_2. \tag{37}$$

While the vorticity transport model is attractive due to its better prediction capabilities in the heat-transfer case, these obvious analytic complications suggested the use of the eddy viscosity model.

The Prandtl-Tollmien hypothesis of constant shear stress across the width of the jet turns out to be of the same form as equation (33). The difference is in mixing length

$$L_{\text{Taylor}} = (\sqrt{2})L_{\text{Prandtl}} \quad \text{or} \quad C_{X\text{Taylor}} = (\sqrt{2})C_{X\text{Prandtl}}.$$

In time, t , the total mass of dust into the collector at the base of the jet may be calculated as

$$W = \rho V \bar{u} A \theta'(\xi_{\text{max}} - \xi_{\text{min}})t \tag{38}$$

where W is the collected dust mass, ρ is the density of the dust, V is the average volume of the particles of dust, \bar{u} is the average velocity of the jet at the collector, A is the collector area, and θ' is the average value of θ in the jet (≈ 0.5 for the fully developed case).

The velocity and concentration profiles are analytically described by functions that are asymptotic in η . Therefore the edge of the jet cannot be clearly defined. If we arbitrarily chose the point where the velocity is 5 per cent of the velocity at the centerline, the Goertler model gives a "jet angle" of approximately 10° in the fully developed region corresponding to a value of $\eta \approx 1.8$. In this region our model estimates a concentration level $\theta \approx 0.10$ for $Sch_T = 0.5$ and $\theta \approx 0.01$ for $Sch_T = 1.0$.

EXPERIMENTS

The apparatus used consisted of a turbulent jet acting across a window of a chamber with provisions for dust generation inside the chamber. The window

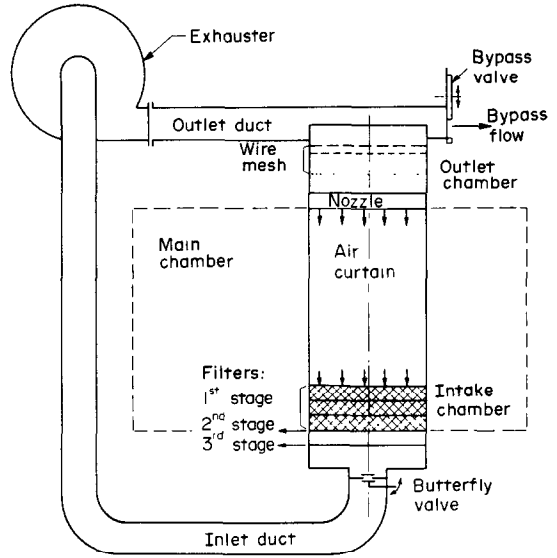


FIG. 3. Air flow in experimental apparatus.

was 12 in wide and 30 in high. An exhauster was used to maintain the curtain running in a closed system (Fig. 3). The flow was governed by two variable orifices.

Approximately constant concentration confined within the main chamber was a requirement for successful measurements. The atmosphere inside the chamber was sampled through a Bausch and Lomb automatic particle counter model 40-1A. The concentration was monitored for some time, while the input parameters were adjusted in order to produce minimum fluctuations of concentration. The input parameters for the concentration function are: curtain characteristic velocity and dust generator settings such as pressure of the dust agitating pulse, pulse width and frequency.

The dust generator shown in Fig. 4 was the device employed to provide dust flux into the chamber.

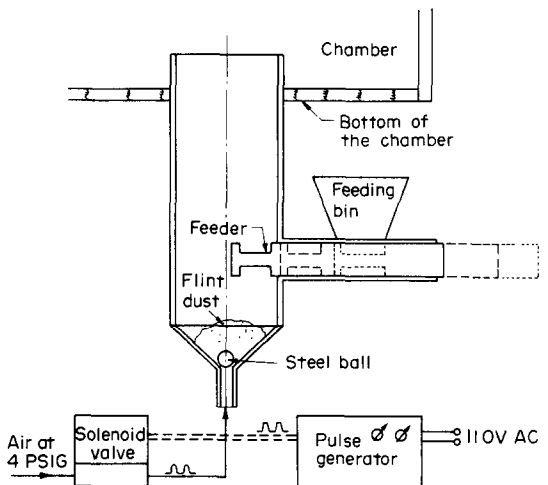


FIG. 4. Dust generator.

Pressurized air at 3–5 psig was used to agitate the powder by means of a solenoid valve regulated by a variable pulse generator. The typical pulse frequency was about 0.05 Hz and pulse width, 0.2 s. The concentration in the chamber could not be brought to a completely steady state, however it decayed smoothly and slowly over time and by constantly referring to the chamber level, normalized data could be obtained. Details of the procedure are available in Bednarik [5]. Flint dust was chosen as a suitable dust, having essentially nontoxic properties and a size distribution suitable for this study. The range under $10\ \mu\text{m}$ is generally considered the inhalable range. The size distribution of flint is shown in Fig. 5. The particles

was measured at different L/b_0 in both directions from the centerline, in order to assure that the jet had the velocity profile and the centerline velocity of the classical theoretical calculations for two-dimensional jets. A set of five check points was chosen close to the inlet grille and the butterfly valve was adjusted until the pressure readings on the check points were showing values close to the expected figures. The final jet outlet velocity was 28.6 ft/s. A set of velocity profile measurements was made at three different L/b_0 values of 15, 30 and 45. These are shown in Fig. 6. The centerline velocity measurement is shown in Fig. 7. The variation of velocity away from the center of the curtain was also checked and the flow was unvarying over a

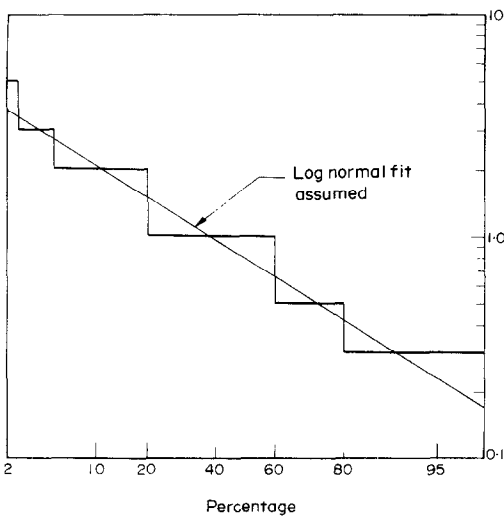


FIG. 5. Flint size distribution.

are irregular with few acute surface angles, making particle sizing straightforward. The maximum particle concentration was 10^7 particles/cm³ in any test. The volume mean diameter was $0.3\ \mu\text{m}$ and the specific gravity of flint is 2.65. This results in a maximum difference in density between pure air and the aerosol of 0.03 per cent.

Due to the fact that the air curtain is a recirculating system, the intake duct was equipped with a three stage filter in order to prevent cumulative contamination of the curtain air. The last filter stage was made out of the type of filter material used for environmental air pollution experiments and its rating is 99.9 per cent of particles of $0.2\ \mu$ in size.

Due to suction at the inlet grille of the filtering chamber, care had to be taken to preserve the classical two-dimensional nature of the jet. With the outlet velocity selected at 30 ft/s, giving a Reynolds number, based on exit width of $Re = 4160$, the velocity profile

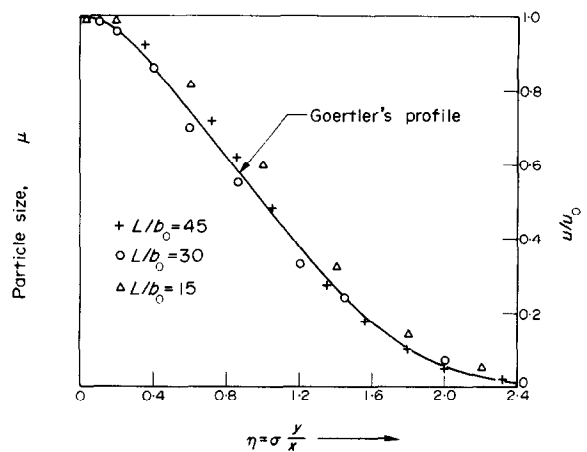


FIG. 6. Velocity profile for two-dimension jet.

minimum of three jet widths for $L/b_0 = 45$. The longitudinal axis of the jet was kept from bending by means of a 3 in I.D. vent located in one corner of the chamber, which equalized the static pressure on both sides of the air curtain.

The particle concentration was measured at four different values of L/b_0 across the full depth of the jet. The method for concentration evaluation made use of the automatic particle counter. The flow input to the counter was 170 cm³/min. The probe was fastened to a two-dimensional traverse mechanism and a standard pitot tube was fastened parallel to the probe at identical elevation in order to make identification of the jet centerline possible.

Figure 8 shows typical samples across the jet. The data were taken at four different elevations, L/b_0 , and a minimum of five traverses were taken at each L/b_0 elevation, starting at $L/b_0 = 40$ and progressing upwards. The data points corrected for the drift or

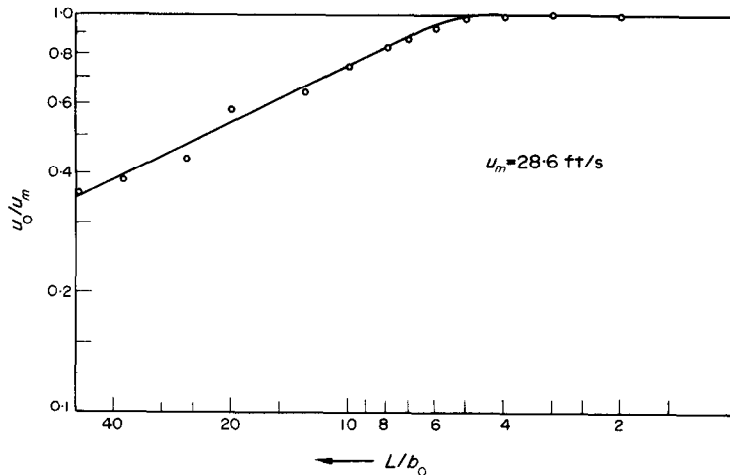


FIG. 7. Decay of centerline velocity.

reference concentration level in the anchor were reproducible within an accuracy of approximately 7 per cent.

When the velocity in the sampling probe is different from that in the measured stream of gas, some discrepancy of results can be expected. Watson [6] specifies that the error for particles less than about 5μ in size, for ordinary sampling velocities, such as those used in this experiment, is insignificant.

DISCUSSION

The concentration profile for $L/b_0 = 14$ is shown in Fig. 8(a). It is obvious that the agreement with the analytic model is poor. The flow in this region is not a fully developed jet but has the properties of a mixing layer as far as particle concentration is concerned. The analytic solution for the mixing layer is after Abramovich [1]. Also this data is in qualitative agreement with the finite difference analysis of Hayes [4] for temperature distributions (see Fig. 2). It can be concluded that the full development of the concentration profile takes place between approximately $0 < L/b_0 < 20$. This is generally indicated for both temperature and concentration gradients. Hayes' finite difference technique evaluated the temperature profile for $0 < L/b_0 < 20$. Both this and his data for temperature gradients indicate this relatively slow approach to "full development".

Variation of the turbulent Schmidt number with particle size was expected due to anticipated change of diffusivity for different particle sizes. The experimental results do not show any orderly variations, which could support this argument in the fully developed region. Hence it was assumed that the difference of the turbulent Schmidt number for $0.5\mu < \text{particle size} < 10\mu$ is insignificant for $L/b_0 > 20$. However, it seems that the smaller particles may

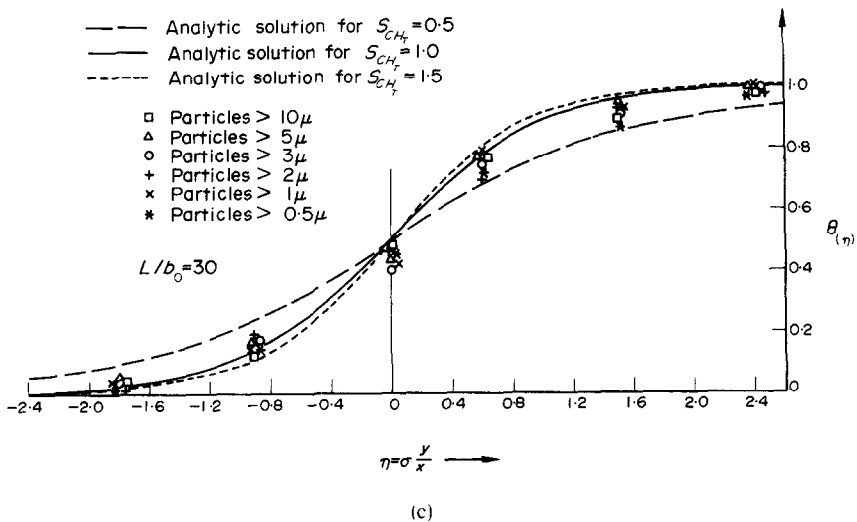
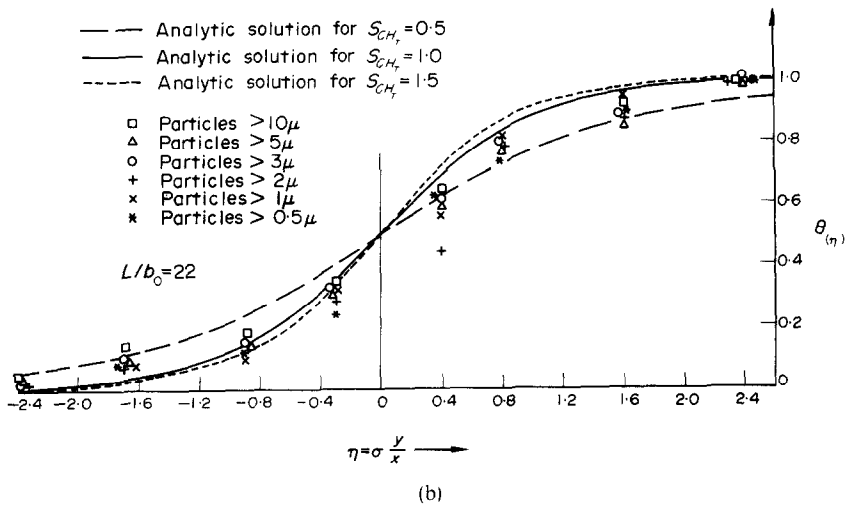
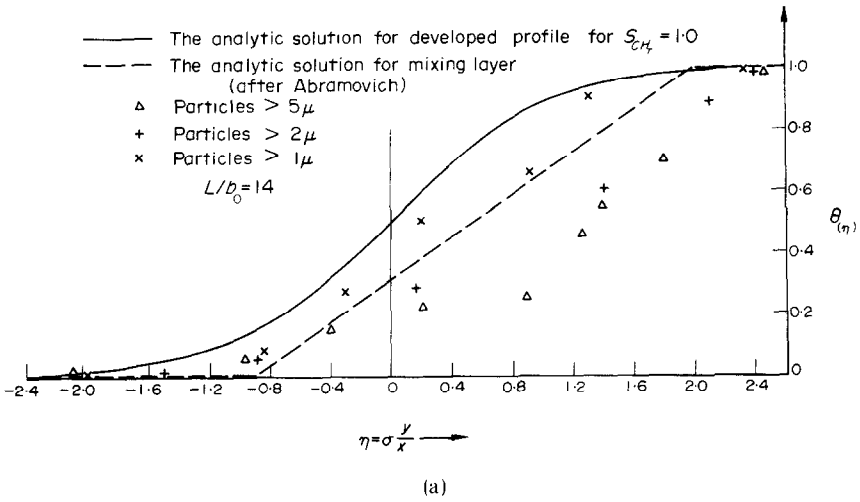
assume the fully developed state sooner than the larger particles, which indicates a change in Schmidt number in the developing region.

CONCLUSIONS

The primary objectives of this study were concerned with the determination of the particle transfer characteristics across a turbulent air curtain by analytic and experimental techniques. The analytic model has been constructed and compared with experiments. The agreement seems to hold for the fully developed region only, which for the concentration profile means L/b_0 greater than about 20. This is larger than the value for velocity profile development which is about 6.

The analysis based on constant eddy viscosity was solved for steady state conditions at zero pressure drop across the jet. The experimental data suggests a Schmidt number ≈ 0.5 . The expected change of the Schmidt number with the particle size was not found within the precision of the experimental procedure and for particles $\leq 10\mu$. The concentration profile is universal for the fully developed jet for $L/b_0 > 20$. The section of the jet $0 \leq L/b_0 \leq 6$ is governed by the models for mixing zone discussed by other workers. In addition, the section $6 < L/b_0 < 20$ does not seem to fit the mixing region theory or the fully developed concentration profile described in this study. It is therefore concluded that the tendency to assume the fully developed state is much slower for particle concentration than for the velocity profile.

The phenomenon described in this study might have several engineering applications. It might be possible for instance to arrange for isolation of places with concentrations, aerosols, or gases which are environmental or health hazards. This can be done by installation of a return type air curtain with filtered intake.



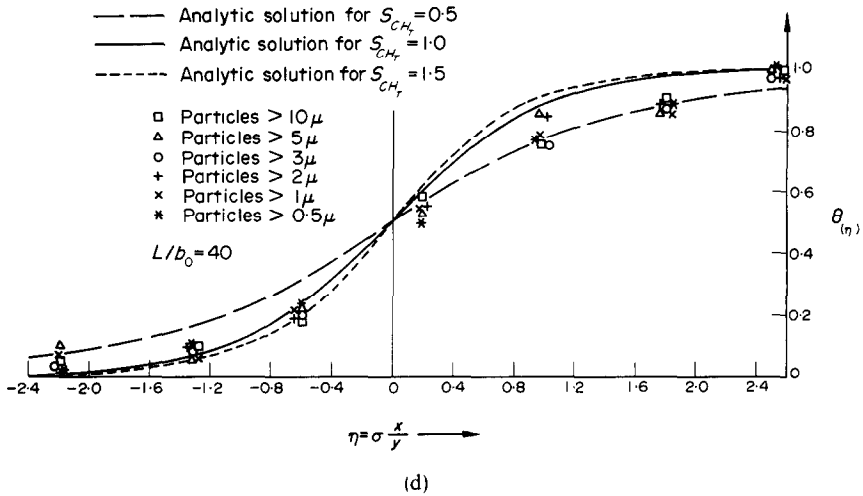


FIG. 8. Concentration profiles for (a) $L/b_0 = 14$, (b) $L/b_0 = 22$, (c) $L/b_0 = 30$, (d) $L/b_0 = 40$.

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